VALUE AT RISK

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ADVERTENCIA:

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TODOS LOS MODELOS ESTÁN MAL. . .

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ADVERTENCIA:

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...ALGUNOS SON ÚTILES.

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Riesgo

Riesgo $\Rightarrow$ Incertidumbre

Riesgo $\Rightarrow$ Incertidumbre $\Rightarrow$ Probabilidad

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Cuantificar riesgo

Riesgo $\Rightarrow$ Incertidumbre $\Rightarrow$ Probabilidad

Cuantificar riesgo $\longrightarrow$ **Variable aleatoria**

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Cuantificar riesgo $\longrightarrow$ **Variable aleatoria**

$$\mathbf{X} : \Omega \longrightarrow \mathbb{R}$$

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$$F(x) = \mathbb{P}(\mathbf{X} \leq x)$$

$$\mathbf{X} : \Omega \longrightarrow \mathbb{R}$$

$$F(x) = \mathbb{P}(\mathbf{X} \leq x)$$

$$\mathbb{P}(a < \mathbf{X} \leq b) = F(b) - F(a)$$

Riesgo 1

Riesgo 1 $\longrightarrow$

Riesgo 1 $\longrightarrow$ $\mathbf{X}_1$

$$\text{Riesgo 1} \quad \longrightarrow \quad \mathbf{X}_1 \quad \sim \quad F_1$$

$$\text{Riesgo 1} \longrightarrow \mathbf{X}_1 \sim F_1$$
$$\text{Riesgo 2} \longrightarrow \mathbf{X}_2 \sim F_2$$

$$
\begin{array}{ccccc}
\text{Riesgo } 1 & \longrightarrow & \mathbf{X}_1 & \sim & F_1 \\
\text{Riesgo } 2 & \longrightarrow & \mathbf{X}_2 & \sim & F_2 \\
\vdots & \vdots & & \vdots & \\
\text{Riesgo } n & \longrightarrow & \mathbf{X}_n & \sim & F_n
\end{array}
$$

$$\begin{array}{lcccc}
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\vdots & \vdots & & \vdots & \\
\text{Riesgo } n & \longrightarrow & \mathbf{X}_n & \sim & F_n
\end{array}$$

- -

Total

$$
\begin{array}{lcccc}
\text{Riesgo } 1 & \longrightarrow & \mathbf{X}_1 & \sim & F_1 \\
\text{Riesgo } 2 & \longrightarrow & \mathbf{X}_2 & \sim & F_2 \\
\vdots & \vdots & & \vdots & \\
\text{Riesgo } n & \longrightarrow & \mathbf{X}_n & \sim & F_n
\end{array}
$$

- -

$$
\textbf{Total} \;\rightarrow\; \mathbf{S} = \mathbf{X}_1 + \cdots + \mathbf{X}_n \;\sim\; \mathbf{?}
$$

Total

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$$(\,\mathbf{X}_1\,,\,\mathbf{X}_2\,,\,\ldots\,,\,\mathbf{X}_n\,) \;\sim\; H(x_1, x_2, \ldots, x_n)$$

$$\textbf{Total} \quad \longrightarrow \quad \mathbf{S} \; = \; \mathbf{X}_1 + \cdots + \mathbf{X}_n \quad \sim \quad \textbf{?}$$

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$$H \quad \sim \quad ?$$

$$\text{Riesgo} \quad \longrightarrow \quad \text{v.a. } \mathbf{X}$$

Riesgo $\longrightarrow$ v.a. $\mathbf{X}$

Medida de riesgo:

Riesgo $\longrightarrow$ v.a. $\mathbf{X}$

Medida de riesgo: $\varrho : \mathbf{X} \to \varrho(\mathbf{X})$

Riesgo $\longrightarrow$ v.a. $\mathbf{X}$

Medida de riesgo: $\varrho : \mathbf{X} \rightarrow \varrho(\mathbf{X})$

$ adicional $\longrightarrow$ **Riesgo aceptable**

Monotonicidad

Monotonicidad

Si $\mathbf{X}_1 \leq \mathbf{X}_2$

Monotonicidad

$$\text{Si} \quad \mathbf{X}_1 \leq \mathbf{X}_2 \quad \Rightarrow \quad \varrho(\mathbf{X}_1) \leq \varrho(\mathbf{X}_2)$$

Traslación

Traslación

$$\varrho(\mathbf{X} + \text{constante})$$

Traslación

$$\varrho(\mathbf{X} + \text{constante}) = \varrho(\mathbf{X}) + \text{constante}$$

Subaditividad

Subaditividad

$$\varrho(\mathbf{X}_1 + \mathbf{X}_2)$$

Subaditividad

$$\varrho(\mathbf{X}_1 + \mathbf{X}_2) \quad \leq \varrho(\mathbf{X}_1) + \varrho(\mathbf{X}_2)$$

Subaditividad

$$\varrho(\mathbf{X}_1 + \mathbf{X}_2) \quad \leq \varrho(\mathbf{X}_1) + \varrho(\mathbf{X}_2)$$

"La diversificación, si acaso disminuye el riesgo, no lo incrementa"

Subaditividad

$$\varrho(\mathbf{X}_1 + \mathbf{X}_2) \leq \varrho(\mathbf{X}_1) + \varrho(\mathbf{X}_2)$$

"La diversificación, si acaso disminuye el riesgo, no lo incrementa" **?!**

Homogeneidad positiva

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$$k > 0$$

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$$k > 0 : \qquad \varrho(k\mathbf{X}) = k\varrho(\mathbf{X})$$

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$\varrho(n\mathbf{X})$

Homogeneidad positiva

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$$\varrho(n\mathbf{X}) = \varrho(\mathbf{X} + \mathbf{X} + \cdots + \mathbf{X})$$

Homogeneidad positiva

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$$\varrho(n\mathbf{X}) = \varrho(\mathbf{X} + \mathbf{X} + \cdots + \mathbf{X}) \leq n\varrho(\mathbf{X})$$

Homogeneidad positiva

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$$\varrho(n\mathbf{X}) = \varrho(\mathbf{X} + \mathbf{X} + \cdots + \mathbf{X}) = n\varrho(\mathbf{X})$$

Value at Risk

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Riesgo $\mathbf{X} \sim F(x)$

Value at Risk

Riesgo $\mathbf{X} \sim F(x) \qquad 0 < \alpha < 1$

Value at Risk

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$$\text{VaR}_\alpha(\mathbf{X}) = x_0$$

Value at Risk

Riesgo $\mathbf{X} \sim F(x) \qquad 0 < \alpha < 1$

$$\text{VaR}_\alpha(\mathbf{X}) = x_0 \qquad \mathbb{P}(\mathbf{X} \leq x_0) = \alpha$$

Value at Risk

Riesgo $\mathbf{X} \sim F(x)$ $\qquad 0 < \alpha < 1$

$$\mathrm{VaR}_{\alpha}(\mathbf{X}) = x_0 \qquad \mathbb{P}(\mathbf{X} \leq x_0) = \alpha$$

$$F(x_0) = \alpha \quad \Rightarrow \quad x_0 = F^{-1}(\alpha)$$

Value at Risk

Value at Risk

- No es subaditiva

Value at Risk

- No es subaditiva: puede ocurrir que

$$\mathrm{VaR}(\mathbf{X}_1 + \mathbf{X}_2) > \mathrm{VaR}(\mathbf{X}_1) + \mathrm{VaR}(\mathbf{X}_2)$$

Value at Risk

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$$\mathrm{VaR}(\mathbf{X}_1 + \mathbf{X}_2) > \mathrm{VaR}(\mathbf{X}_1) + \mathrm{VaR}(\mathbf{X}_2)$$

- Si ocurre $\mathbf{X} > \mathrm{VaR}(\mathbf{X}) \ldots$

Value at Risk

- No es subaditiva: puede ocurrir que

$$\mathrm{VaR}(\mathbf{X}_1 + \mathbf{X}_2) > \mathrm{VaR}(\mathbf{X}_1) + \mathrm{VaR}(\mathbf{X}_2)$$

- Si ocurre $\mathbf{X} > \mathrm{VaR}(\mathbf{X}) \ldots \mathbf{X} \approx ?$

Tail VaR

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$$\text{TVaR}_\alpha(\mathbf{X}) \quad = \quad \mathbb{E}\left(\mathbf{X} \,|\, \mathbf{X} > \text{VaR}_\alpha(\mathbf{X})\right)$$

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- Es medida *coherente* de riesgo

Tail VaR

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- Es medida *coherente* de riesgo
- Puede ocurrir que. . .

Tail VaR

$$\mathrm{TVaR}_\alpha(\mathbf{X}) = \mathbb{E}\,(\mathbf{X} \,|\, \mathbf{X} > \mathrm{VaR}_\alpha(\mathbf{X}))$$

- Es medida *coherente* de riesgo
- Puede ocurrir que... $\mathrm{TVaR}(\mathbf{X}) = +\infty$ **?!**

Tail Median VaR

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$$\text{TMVaR}_\alpha(\mathbf{X}) = \mathbb{M}(\mathbf{X} \,|\, \mathbf{X} > \text{VaR}_\alpha(\mathbf{X}))$$

Tail Median VaR

$$\begin{aligned}\mathrm{TMVaR}_\alpha(\mathbf{X}) &= \mathbb{M}\left(\mathbf{X} \,|\, \mathbf{X} > \mathrm{VaR}_\alpha(\mathbf{X})\right) \\ &= \mathrm{VaR}_{(1+\alpha)/2}(\mathbf{X})\end{aligned}$$

Tail Median VaR

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- No es subaditiva

Tail Median VaR

$$\begin{aligned} \text{TMVaR}_\alpha(\mathbf{X}) &= \mathbb{M}\,(\mathbf{X} \,|\, \mathbf{X} > \text{VaR}_\alpha(\mathbf{X})) \\ &= \text{VaR}_{(1+\alpha)/2}(\mathbf{X}) \end{aligned}$$

- No es subaditiva
- Nunca es $+\infty$

$$\mathbf{S} = \mathbf{X}_1 + \cdots + \mathbf{X}_n$$

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$$\varrho(\mathbf{S}) = \mathbf{?}$$

$$\mathbf{S} \;=\; \mathbf{X}_1 + \cdots + \mathbf{X}_n$$

$$\varrho(\mathbf{S}) \;=\; \mathbf{?} \qquad\qquad \mathbf{S} \;\sim\; \mathbf{?}$$

$$\mathbf{S} = \mathbf{X}_1 + \cdots + \mathbf{X}_n$$

$$\varrho(\mathbf{S}) = \mathbf{?} \qquad\qquad \mathbf{S} \sim \mathbf{?}$$

$$(\, \mathbf{X}_1 \,,\, \mathbf{X}_2 \,,\, \ldots \,,\, \mathbf{X}_n \,) \sim H(x_1, x_2, \ldots, x_n)$$

$$\mathbf{S} = \mathbf{X}_1 + \cdots + \mathbf{X}_n$$

$$\varrho(\mathbf{S}) = \mathbf{?} \qquad\qquad \mathbf{S} \sim \mathbf{?}$$

$$(\,\mathbf{X}_1\,,\,\mathbf{X}_2\,,\,\ldots\,,\,\mathbf{X}_n\,) \sim H(x_1, x_2, \ldots, x_n)$$

$$H \sim \mathbf{?}$$

Función cópula

$$\mathbf{X}_1 \sim F_1(x_1)$$

$$\mathbf{X}_1 \sim F_1(x_1) \qquad \mathbf{X}_2 \sim F_1(x_2)$$

$$\mathbf{X}_1 \sim F_1(x_1) \qquad \mathbf{X}_2 \sim F_1(x_2)$$

$$(\mathbf{X}_1, \mathbf{X}_2) \sim H(x_1, x_2)$$

$$\mathbf{X}_1 \sim F_1(x_1) \qquad \mathbf{X}_2 \sim F_1(x_2)$$

$$(\mathbf{X}_1, \mathbf{X}_2) \sim H(x_1, x_2)$$

$$H(x_1, x_2) = \mathbf{C}\big(F_1(x_1) , F_2(x_2) \big)$$

$$H(x_1, x_2) = \mathbf{C}\big(\, F_1(x_1)\,,\; F_2(x_2)\,\big)$$

$$H(x_1, x_2) = \mathbf{C}\big(F_1(x_1)\,,\; F_2(x_2) \big)$$

C

$$H(x_1, x_2) = \mathbf{C}\big(F_1(x_1), F_2(x_2)\big)$$

$$\mathbf{C} \longrightarrow \text{Dependencia}$$

$$H(x_1, x_2) = \mathbf{C}\big(\, F_1(x_1)\,,\; F_2(x_2)\,\big)$$

$$\mathbf{C} \quad \longrightarrow \quad \text{Dependencia}$$

$F_1\,,\; F_2$

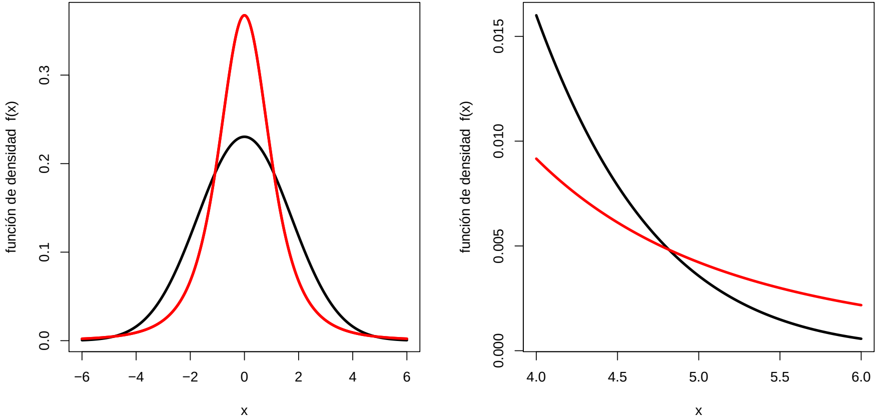
$$H(x_1, x_2) = \mathbf{C}\big(F_1(x_1)\, ,\; F_2(x_2) \big)$$

$$\mathbf{C} \longrightarrow \text{Dependencia}$$

$$F_1\, ,\; F_2 \longrightarrow \text{Comportamiento individual}$$

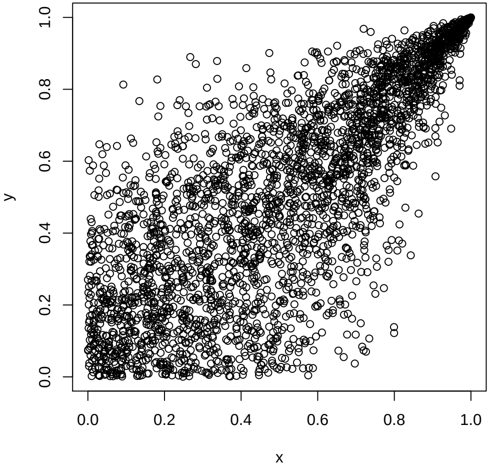
Cola más pesada que la Normal

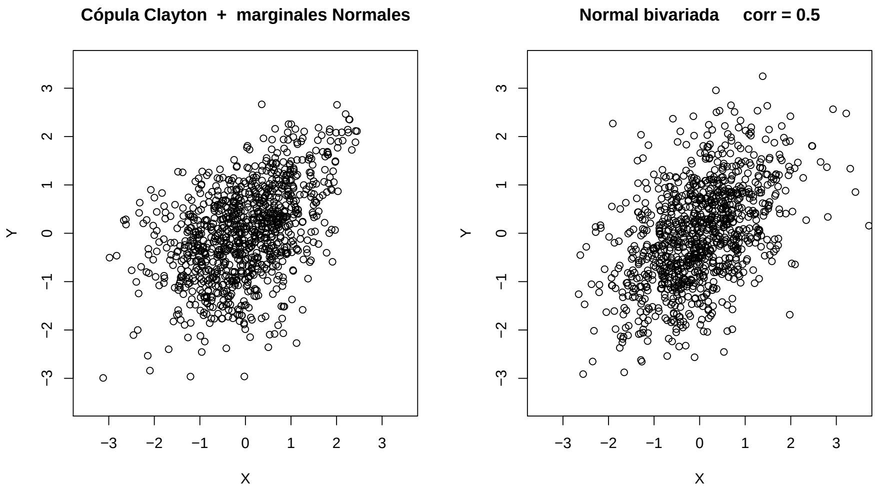
Cola más pesada que la Normal



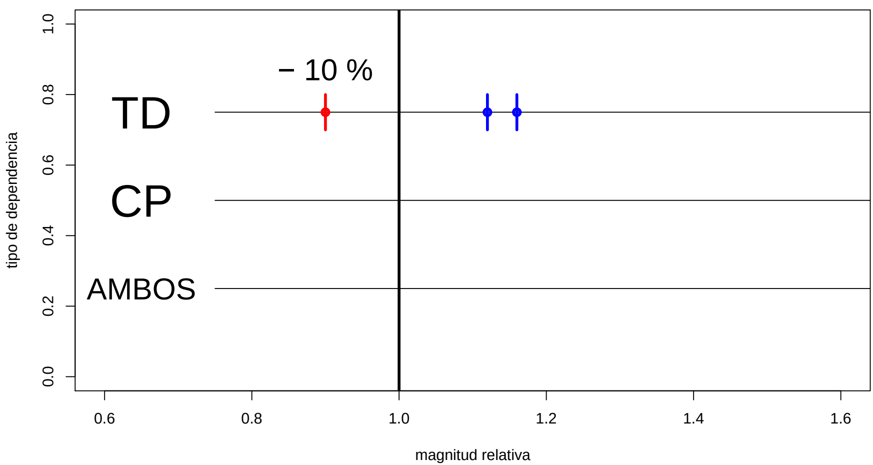
Tail dependence

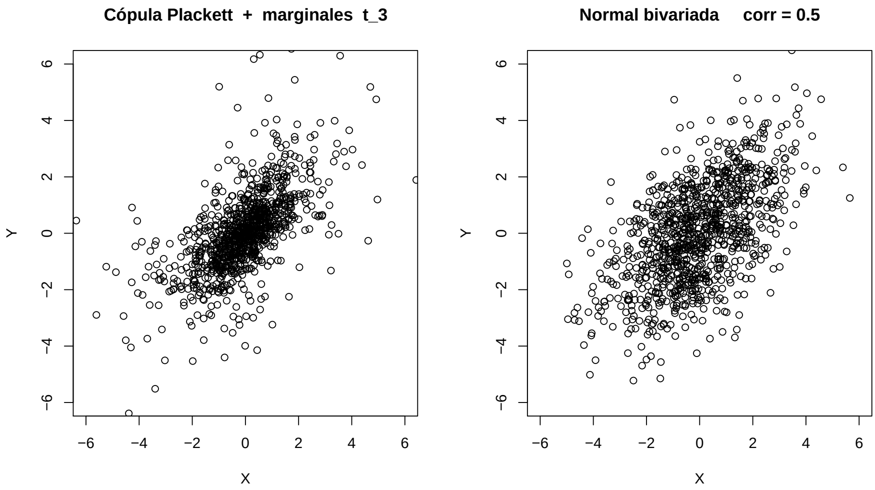
Tail dependence



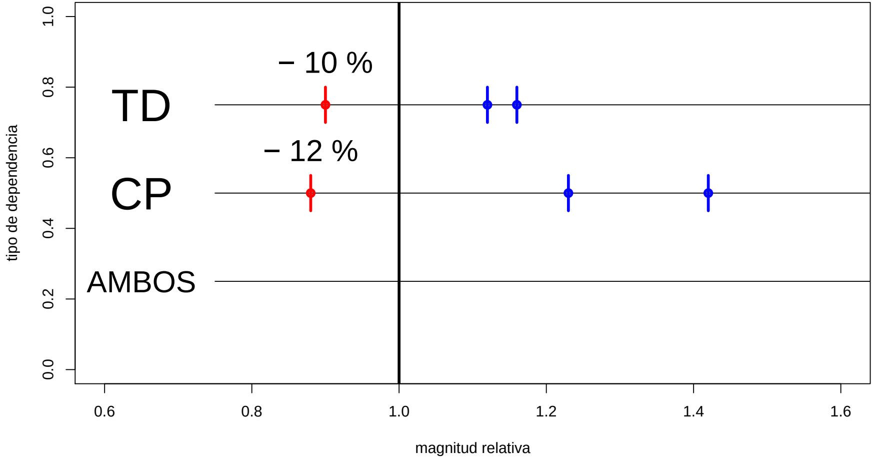

Cópula Clayton + marginales Normales
Normal bivariada corr = 0.5
Y
X
−3
−2
−1
0
1
2
3

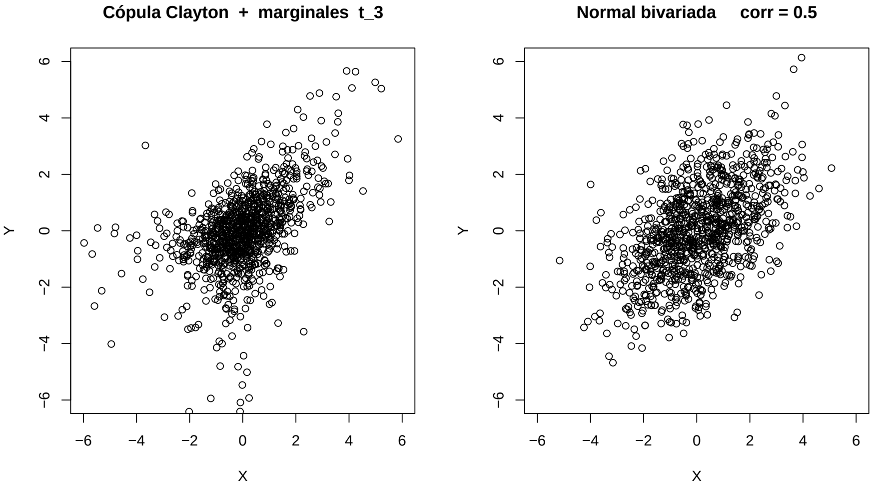
VaR Normal | VaR (real) | TMVaR | TVaR

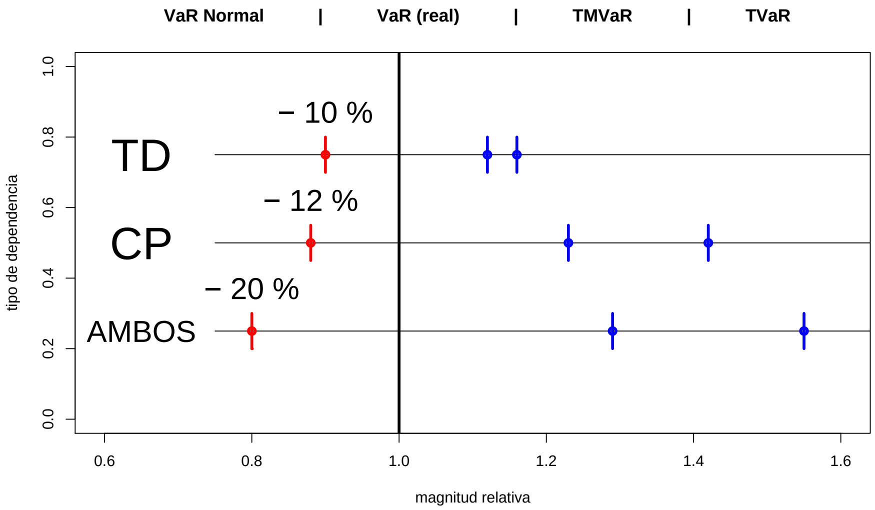



Cópula Plackett + marginales t_3
Normal bivariada corr = 0.5
X
Y
−6
−4
−2
0
2
4
6

VaR Normal | VaR (real) | TMVaR | TVaR




Cópula Clayton + marginales t_3
Normal bivariada corr = 0.5
X
Y
−6
−4
−2
0
2
4
6


VaR Normal | VaR (real) | TMVaR | TVaR
TD
CP
AMBOS
− 10 %
− 12 %
− 20 %
tipo de dependencia
magnitud relativa
0.0
0.2
0.4
0.6
0.8
1.0
0.6
0.8
1.0
1.2
1.4
1.6

GRACIAS

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